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# Money's Role in the Monetary Business Cycle

A small, structural model of the monetary business cycle implies that real money balances enter into a correctly-specified, forward-looking IS curve if and only if they enter into a correctly-specified, forward-looking Phillips curve. The model also implies that empirical measures of real balances must be adjusted for shifts in money demand to accurately isolate and quantify the dynamic effects of money on output and inflation. Maximum likelihood estimates of the model's parameters take both these considerations into account, but still suggest that money plays a minimal role in the monetary business cycle.

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FOR MACROECONOMISTS, recent years have been ones of heightened interest in monetary aspects of the business cycle. Analysts have devoted considerable time and effort towards developing new and improved models for monetary policy evaluation.

These newly-developed models—ranging from Rotemberg and Woodford (1997) and McCallum and Nelson's (1999) forward-looking models with microfoundations to Fuhrer and Moore's (1995) forward-looking model without microfoundations to Rudebusch and Svensson's (2002) backward-looking model without microfoundations—differ considerably in their details. One feature that is shared by all of these models, however, has to do with the minimal role that each assigns to changes in the stock of money. The Rotemberg–Woodford and Fuhrer–Moore models, for

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instance, make no reference whatsoever to any of the monetary aggregates. The McCallum–Nelson and Rudebusch–Svensson models include money, but only through a money demand equation that ultimately serves to determine how much money need be supplied, given the levels of output, inflation, and interest rates. In none of these models do changes in the nominal or real quantities of money play a direct role in shaping the dynamic behavior of other variables.

Do these existing models provide an accurate and complete description of money's role in the monetary business cycle? That is the question addressed here.

The paper begins, in Section 1 below, by constructing a small, structural model with microfoundations that allows, but does not require, changes in the real money stock to directly affect the dynamics of output and inflation. It goes on, in Section 2, to estimate the model with quarterly time-series data from the post-1980 U.S. economy and to assess the statistical adequacy of popular specifications that assign money a minimal role.

The theoretical and empirical analyses yield several insights. First, the theoretical model contains three equations summarizing the optimizing behavior of the households and firms that populate the economy. The first of these resembles the IS curve in traditional Keynesian models, the second takes the form of a money demand relationship, and the third is a forward-looking version of the Phillips curve. Of course, real balances always enter into the money demand function. But the cross-equation restrictions imposed by the model imply that real balances enter into the IS curve if and only if they also enter into the Phillips curve specification. Thus, according to the theory, if changes in the real stock of money have a direct impact on the dynamics of output and inflation, then that impact must come simultaneously through *both* the IS and the Phillips curve relationships.

Second, the model reveals that assessing the importance of real balances in the forward-looking IS and Phillips curve specifications involves more than simply adding some measure of money to the equations and testing for the statistical significance of the associated coefficients. To isolate and quantify the effects of changes in real balances on output and inflation, the measure of money must be adjusted for shifts in money demand. Some clear intuition underlies this result. Suppose, following Taylor (1993), that actual Federal Reserve policy is best described as one that manages the short-term interest rate rather than one of the monetary aggregates. Poole's (1970) analysis of a traditional Keynesian model suggests that such a policy works to insulate the economy from the effects of money demand disturbances, and Ireland (2000) shows that Poole's results carry over to a forward-looking, microfounded model like the one used here. Taken together, these studies imply that by managing short-term interest rates, Federal Reserve policy gives rise to changes in the money supply that simply accommodate changes in money demand, leaving output and inflation unchanged. Thus, a measure of real balances that is not corrected for shifts in money demand may seem unrelated to output and inflation, even when it is possible for completely exogenous changes in money to have important direct effects on both variables.

Third, and finally, the empirical analysis reveals that once the cross-equation restrictions and the money demand disturbances identified by the theoretical model are accounted for, the post-1980 U.S. data seem to prefer the standard specification, in which real balances are absent from the IS and Phillips curves. Evidently, previous studies are justified in their minimal treatment of money's role in the monetary business cycle.

## 1. A SMALL, STRUCTURAL MODEL OF THE MONETARY BUSINESS CYCLE

Here, the small, structural model developed in Ireland (1997, 2000) is modified to focus on the direct effects that changes in real money balances may have on the dynamics of output and inflation. This model elaborates on Rotemberg's (1982) framework, in which monopolistically competitive firms face a quadratic cost of nominal price adjustment. Relative to the model in Ireland (1997, 2000), the one used here is generalized by allowing, but not requiring, real balances to appear in the IS and Phillips curve specifications. At the same time, however, the model is simplified, following Rotemberg and Woodford (1997), by abstracting from the process of capital accumulation.

The economy consists of a representative household, a representative finished goods-producing firm, a continuum of intermediate goods-producing firms indexed by $i \in [0,1]$, and a monetary authority. During each period $t = 0,1,2,...$, each intermediate goods-producing firm produces a distinct, perishable intermediate good. Hence, intermediate goods may also be indexed by $i \in [0,1]$, where firm $i$ produces good $i$. The model features enough symmetry, however, to allow the analysis to focus on the behavior of a representative intermediate goods-producing firm that produces the generic intermediate good $i$.

The representative household seeks to maximize the expected utility function,

$$E\sum_{t=0}^{\infty} \beta^t a_t\{u[c_t, (M_t/P_t)/e_t] - \eta h_t\},$$

with $1 > \beta > 0$ and $\eta > 0$, subject to the budget constraint

$$M_{t-1} + T_t + B_{t-1} + W_t h_t + D_t \geq P_t c_t + B_t/r_t + M_t,$$

which must be satisfied for all $t = 0,1,2,....$ In the utility function, $c_t$, $M_t/P_t$, and $h_t$ denote the household's consumption, real balances, and labor supply during period $t$. The preference shocks $a_t$ and $e_t$ follow the autoregressive processes

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_{at} \tag{1}$$

and

$$\ln(e_t) = (1 - \rho_e)\ln(e) + \rho_e \ln(e_{t-1}) + \varepsilon_{et}, \tag{2}$$

where $1 > \rho_a > 0$, $1 > \rho_e > 0$, $e > 0$, and the zero-mean, serially uncorrelated innovations $\varepsilon_{at}$ and $\varepsilon_{et}$ are normally distributed with standard deviations $\sigma_a$ and $\sigma_e$. As shown below, the shocks $a_t$ and $e_t$ translate, in equilibrium, into disturbances to the model's IS and money demand curves.

In the budget constraint, the household's sources of funds include $M_{t-1}$, nominal money carried into period $t$, $T_t$, a lump-sum nominal transfer received from the monetary authority at the beginning of period $t$, and $B_{t-1}$, the value of nominal bonds maturing during period $t$. The household's sources of funds also include labor income, $W_t h_t$, where $W_t$ denotes the nominal wage, and nominal dividend payments, $D_t$, received from the intermediate goods-producing firms. The household's uses of funds consist of consumption, $c_t$, of the finished good, purchased at the nominal price, $P_t$, newly-issued bonds of value $B_t/r_t$, where $r_t$ denotes the gross nominal interest rate, and the money, $M_t$, to be carried into period $t+1$. It is convenient in what follows to let $m_t = M_t/P_t$ denote the household's real balances and $\pi_t = P_t/P_{t-1}$ denote the gross inflation rate during period $t$.

During each period $t = 0,1,2,...$, the representative finished goods-producing firm uses $y_t(i)$ units of each intermediate good $i \in [0,1]$, purchased at the nominal price, $P_t(i)$, to manufacture $y_t$ units of the finished good according to the constant-returns-to-scale technology described by

$$\left[\int_0^1 y_t(i)^{(\theta-1)/\theta}\, di\right]^{\theta/(\theta-1)} \geq y_t\,,$$

with $\theta > 1$. The finished goods-producing firm maximizes its profits by choosing

$$y_t(i) = [P_t(i)/P_t]^{-\theta} y_t\,,$$

which reveals that $\theta$ measures the constant price elasticity of demand for each intermediate good. Competition drives the finished goods-producing firm's profits to zero in equilibrium, determining $P_t$ as

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di\right]^{1/(1-\theta)}.$$

During each period $t = 0,1,2,...$, the representative intermediate goods-producing firm hires $h_t(i)$ units of labor from the representative household to manufacture $y_t(i)$ units of intermediate good $i$ according to the linear technology

$$z_t h_t(i) \geq y_t(i).$$

The aggregate productivity shock, $z_t$, follows the autoregressive process

$$\ln(z_t) = (1-\rho_z)\ln(z) + \rho_z \ln(z_{t-1}) + \varepsilon_{zt}\,, \tag{3}$$

where $1 > \rho_z > 0$, $z > 0$, and the zero-mean, serially uncorrelated innovation, $\varepsilon_{zt}$, is normally distributed with standard deviation $\sigma_z$. In equilibrium, this supply-side disturbance acts as a shock to the Phillips curve.

Since the intermediate goods substitute imperfectly for one another in producing the finished good, the representative intermediate goods-producing firm sells its output in a monopolistically competitive market: the firm acts as a price-setter, but must satisfy the representative finished goods-producing firm's demand at its chosen price. And here, as in Rotemberg (1982), the intermediate goods-producing firm faces a quadratic cost of adjusting its nominal price, measured in terms of the finished good and given by

$$\frac{\phi}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right]^2 y_t \, ,$$

with $\phi > 0$, where $\pi$ measures the gross steady-state inflation rate. This cost of price adjustment makes the intermediate goods-producing firm's problem dynamic: it chooses $P_t(i)$ for all $t = 0,1,2,...$ to maximize its total market value, as described below in the Appendix. At the end of each period, the firm distributes its profits in the form of a nominal dividend payment, $D_t(i)$, to the representative household.

In a symmetric equilibrium, all intermediate goods-producing firms make identical decisions, so that $y_t(i) = y_t$, $h_t(i) = h_t$, $P_t(i) = P_t$, and $D_t(i) = D_t$ for all $i \in [0,1]$ and $t = 0,1,2,....$ The Appendix shows that in such an equilibrium, the first-order conditions describing the optimizing behavior of the representative household and intermediate goods-producing firm can be approximated by

$$\begin{aligned}\hat{y}_t = E_t\hat{y}_{t+1} - \omega_1(\hat{r}_t - E_t\hat{\pi}_{t+1}) + \omega_2[(\hat{m}_t - \hat{e}_t) \\ - (E_t\hat{m}_{t+1} - E_t\hat{e}_{t+1})] + \omega_1(\hat{a}_t - E_t\hat{a}_{t+1}) \, ,\end{aligned} \tag{4}$$

$$\hat{m}_t = \gamma_1\hat{y}_t - \gamma_2\hat{r}_t + \gamma_3\hat{e}_t \, , \tag{5}$$

and

$$\hat{\pi}_t = (\pi/r)E_t\hat{\pi}_{t+1} + \psi[(1/\omega_1)\hat{y}_t - (\omega_2/\omega_1)(\hat{m}_t - \hat{e}_t) - \hat{z}_t] \, . \tag{6}$$

In Equations (4–6), $\hat{y}_t$, $\hat{m}_t$, $\hat{\pi}_t$, $\hat{r}_t$, $\hat{a}_t$, $\hat{e}_t$, and $\hat{z}_t$ denote the percentage (logarithmic) deviations of $y_t$, $m_t$, $\pi_t$, $r_t$, $a_t$, $e_t$, and $z_t$ from their steady-state values, $y$, $m$, $\pi$, $r$, 1, $e$, and $z$. All the parameters in Equations (4–6) ought to be nonnegative; as shown in the Appendix, each ultimately depends on the underlying parameters describing private agents' tastes and technologies. And in addition to the cross-equation restrictions that appear explicitly in Equations (4–6), the constraints

$$\gamma_1 = (r - 1 + yr\omega_2/m)(\gamma_2/\omega_1) \tag{7}$$

and

$$\gamma_3 = 1 - (r - 1)\gamma_2 \tag{8}$$

must be satisfied.

Equation (4) generalizes McCallum and Nelson's (1999) forward-looking IS curve by allowing real balances $\hat{m}_t$ to enter the specification. The Appendix shows that $\omega_2$ is nonzero, so that the additional terms involving $\hat{m}_t$ are present, if the household's utility function is nonseparable across consumption and real balances. Intuitively, Equation (4) represents a log-linearized version of the Euler equation that links the household's marginal rate of intertemporal substitution to the real interest rate. When utility is nonseparable, real balances affect the marginal rate of intertemporal substitution; hence, they also appear in the IS curve. Equation (5), meanwhile, takes the form of a money demand relationship, with income elasticity $\gamma_1$ and interest semi-elasticity $\gamma_2$.

Equation (6) is a forward-looking Phillips curve that also allows real balances $\hat{m}_t$ to enter the specification when $\omega_2$ is nonzero. According to the model, therefore, real balances belong in a correctly-specified IS curve if and only if they also belong in a correctly-specified Phillips curve. As emphasized by Gali and Gertler (1999) and Sbordone (2002), optimizing firms set prices on the basis of marginal costs; hence, the measure of real economic activity that belongs in a forward-looking Phillips curve such as Equation (6) is a measure of real marginal costs, rather than a measure of detrended output. Here, in this model, real marginal costs depend on real wages, which are in turn linked to the optimizing household's marginal rate of substitution between consumption and leisure. Once again, when utility is nonseparable, real balances affect this marginal rate of substitution; hence, in this case, they also appear in the Phillips curve. Andres, Lopez-Salido, and Valles (2001) show that this result also applies in a model where the nominal price rigidity follows Calvo's (1983) staggering specification instead of the quadratic adjustment cost specification used here.

Equations (4) and (6) also reveal that wherever the real balances variable, $\hat{m}_t$, appears in the IS and Phillips curve relationships, it is followed immediately by the money demand disturbance, $\hat{e}_t$. Thus, according to the model, an empirical measure of real balances must be adjusted for shifts in money demand to obtain an unbiased estimate of the key parameter, $\omega_2$. This shift adjustment becomes particularly important in the case, discussed below, in which the monetary authority accommodates shocks to money demand with offsetting movements in the money supply, for in this case there are potentially large movements in $\hat{m}_t$ that are unrelated to movements in $\hat{m}_t - \hat{e}_t$, $\hat{y}_t$, and $\hat{\pi}_t$.

Seven variables enter into Equations (1–6). Hence, the model is closed by adding a seventh equation—an interest rate rule for monetary policy—of the form

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + \rho_y \hat{y}_{t-1} + \rho_\pi \hat{\pi}_{t-1} + \varepsilon_{rt}\,, \tag{9}$$

where $\rho_r$, $\rho_y$, and $\rho_\pi$ are nonnegative parameters, and where the zero-mean, serially uncorrelated policy shock, $\varepsilon_{rt}$, is normally distributed with standard deviation $\sigma_r$. Like Taylor's (1993) rule, Equation (9) calls for the monetary authority to adjust the short-term nominal interest rate in response to deviations of output and inflation from their steady-state levels. Equation (9) generalizes Taylor's rule, however, by adding a term involving the lagged interest rate: when $\rho_r$ is nonzero, the interest rate adjustment to output and inflation occurs gradually over time.

Ireland (2000) builds on Poole (1970) by showing that in forward-looking, micro-founded models like the one used here, an interest rate rule such as Equation (9) holds the interest rate fixed after a shock to money demand by fully accommodating the shock with a change in the money supply; output and inflation are thereby insulated from the effects of the shock. Under Equation (9), therefore, all movements in $\hat{e}_t$ are mirrored by movements in $\hat{m}_t$; and according to Equations (4) and (6), these offsetting movements contain no information about the magnitude of the key parameter, $\omega_2$. Thus, as suggested above, the shift adjustment of real balances required by Equations (4) and (6) is particularly important in obtaining an unbiased estimate of $\omega_2$.

## 2. ESTIMATION RESULTS

Equations (1–6) and (9) now constitute a system of seven equations in seven variables. The solution to this system, which can be found using the method of Blanchard and Kahn (1980), takes the form of a state-space econometric model. The model's parameters may therefore be estimated by maximum likelihood, as described by Hamilton (1994, chap. 13), using data on four variables: output, real money balances, inflation, and the short-term nominal interest rate.

Thus, in the U.S. data, output is measured by real GDP, real balances are measured by dividing the M2 money stock by the GDP deflator, inflation is measured by changes in the GDP deflator, and the interest rate is measured by the three-month Treasury bill rate. All data, except for the interest rate, are seasonally adjusted. The data for output and real balances are expressed in per-capita terms by dividing by the civilian noninstitutional population, age 16 and over. Distinct upward trends appear in the resulting series for per-capita output and real balances, reflecting the secular growth of the American economy; since the model requires these variables to fluctuate around constant means, a linear trend is removed from the logarithm of each prior to estimation. The data are quarterly and run from 1980:1—also the beginning of Rotemberg and Woodford's (1997) sample—through 2001:3.

Preliminary attempts to estimate all of the model's parameters, described in Ireland (2001), led to unreasonably small values for $\omega_1$ and $\psi$ corresponding, as shown in the Appendix, to extremely high levels of risk aversion and extremely large costs of nominal price adjustment. More sensible results obtain when these two parameters are fixed prior to estimation. Hence, Table 1 displays maximum likelihood estimates of the model's remaining parameters, holding $\omega_1 = 1$ fixed at the value that implies the same level of risk aversion as a utility function that is logarithmic in consumption and holding $\psi = 0.1$ fixed at the value used previously in Ireland (2000). Although 18 parameters appear in Table 1, only 16 of these are estimated independently, since the restrictions Equations (7) and (8) are imposed. The standard errors, also shown in Table 1, correspond to the square roots of the diagonal elements of –1 times the inverted matrix of second derivatives of the maximized log likelihood function.

Before looking specifically at the estimate of $\omega_2$, measuring the importance of real balances in the IS and Phillips curves, it is useful to consider some of the other parameter estimates. In the interest rate rule (Equation 9), the parameter $\rho_y$, measuring

TABLE 1
Maximum Likelihood Estimates and Standard Errors

| Parameter | Unconstrained estimate using M2 | Standard error | Constrained estimate using M2 | Standard error | Unconstrained estimate using M1 | Standard error |
|---|---|---|---|---|---|---|
| $\omega_2$ | 0.0000 | 0.2642 | 0.2500 | — | 0.0000 | 0.2275 |
| $\gamma_1$ | 0.0138 | 0.0767 | 0.0158 | 0.1296 | 0.0030 | 0.0717 |
| $\gamma_2$ | 0.7220 | 0.4967 | 0.1251 | 1.0261 | 0.1592 | 0.8123 |
| $\gamma_3$ | 0.9862 | 0.0108 | 0.9977 | 0.0195 | 0.9970 | 0.0153 |
| $\rho_r$ | 0.5481 | 0.0507 | 0.5481 | 0.0503 | 0.5481 | 0.0508 |
| $\rho_y$ | 0.0000 | 0.0060 | 0.0000 | 0.0059 | 0.0000 | 0.0060 |
| $\rho_\pi$ | 0.5680 | 0.0716 | 0.5680 | 0.0712 | 0.5680 | 0.0716 |
| $\ln(y)$ | 8.8731 | 0.0592 | 8.8747 | 0.0601 | 8.8748 | 0.0587 |
| $\ln(m)$ | 9.7381 | 0.0405 | 9.7383 | 0.0435 | 8.3133 | 0.0696 |
| $\ln(\pi)$ | 0.0102 | 0.0044 | 0.0100 | 0.0046 | 0.0100 | 0.0044 |
| $\ln(r)$ | 0.0189 | 0.0055 | 0.0186 | 0.0057 | 0.0186 | 0.0055 |
| $\rho_a$ | 0.9579 | 0.0302 | 0.9575 | 0.0305 | 0.9575 | 0.0302 |
| $\rho_e$ | 0.9853 | 0.0135 | 0.9867 | 0.0130 | 0.9855 | 0.0131 |
| $\rho_z$ | 0.9903 | 0.0127 | 0.9904 | 0.0126 | 0.9902 | 0.0127 |
| $\sigma_a$ | 0.0188 | 0.0082 | 0.0187 | 0.0082 | 0.0187 | 0.0081 |
| $\sigma_e$ | 0.0088 | 0.0007 | 0.0088 | 0.0007 | 0.0147 | 0.0011 |
| $\sigma_z$ | 0.0098 | 0.0009 | 0.0098 | 0.0008 | 0.0098 | 0.0008 |
| $\sigma_r$ | 0.0025 | 0.0002 | 0.0025 | 0.0002 | 0.0025 | 0.0002 |

the interest rate response to changes in output, is essentially zero, suggesting that Federal Reserve policy has concentrated mainly on controlling inflation during the post-1980 sample period. The estimates $\rho_r = 0.5481$ and $\rho_\pi = 0.5680$ imply a considerable amount of interest rate smoothing and a vigorous policy response to inflation, with a long-run elasticity exceeding unity. This last feature allows the interest rate rule to be consistent with the existence of a unique rational expectations equilibrium, as discussed by Parkin (1978), McCallum (1981), Kerr and King (1996), and Clarida, Gali, and Gertler (2000). Finally, calculations reveal that the relatively small estimate of $\sigma_r = 0.0025$ implies that the model attributes more than 90% of the observed variation in interest rates to the Federal Reserve's deliberate attempts to stabilize inflation in the face of the exogenous demand and supply-side shocks that hit the economy; less than 10% reflects the impact of the policy shock, $\varepsilon_{rt}$.

The estimates of $\ln(y)$, $\ln(m)$, $\ln(\pi)$, and $\ln(r)$ help the model match the average level of each variable in the data. These parameters are estimated in logs, rather than levels, to avoid scaling problems in the numerical routine that maximizes the likelihood function. The estimates of $\rho_a$, $\rho_e$, and $\rho_z$ indicate that the exogenous IS, money demand, and Phillips curve shocks are quite persistent, while the sizable estimates of $\sigma_a$, $\sigma_e$, and $\sigma_z$ imply that all three of these shocks are important in explaining fluctuations in the data.

Turning at last to the key parameter, $\omega_2$, the maximum likelihood estimate is essentially zero, indicating that the data prefer the popular version of the model, in which real balances are completely absent from the IS and Phillips curve specifications. This estimate of $\omega_2$ also provides an explanation for the very small estimate of $\gamma_1$, measuring the income elasticity of money demand, that appears in Table 1. Plugging the calibrated value of $\omega_1 = 1$ and the estimated value of $r = 1.0189$ into

Equation (7) reveals that with $\omega_2 = 0$, the income elasticity, $\gamma_1$, must necessarily be a very small multiple of the interest semi-elasticity parameter, $\gamma_2$. Put another way, these estimation results indicate that the data want to set $\omega_2 = 0$, even though the cost of this choice is an unreasonably small implied value for the income elasticity of money demand.

The standard error associated with the estimate of $\omega_2 = 0$ is quite sizable, however, suggesting that specifications with larger values of this key parameter may do nearly as well in fitting the data. To explore this possibility, Table 1 also displays constrained maximum likelihood estimates, obtained after imposing the restriction that $\omega_2 = 0.25$. With this constraint imposed, the maximized log likelihood falls from 1359.3 to 1357.8: a likelihood ratio test rejects the constrained model in favor of the unconstrained alternative with $\omega_2 = 0$, but only at the 10% level, while the Wald test based on a comparison of the estimate $\omega_2 = 0$ to its standard error fails to reject the constraint at any conventional significance level.

Table 1 shows that when the model is reestimated with $\omega_2 = 0.25$ held fixed, most of the model's other parameters remain unchanged. A notable exception, however, is the estimate of the interest semi-elasticity of money demand, which falls from $\gamma_2 = 0.7220$ when $\omega_2 = 0$ to $\gamma_2 = 0.1251$ when $\omega_2 = 0.25$. In the constrained specification with $\omega_2 = 0.25$, therefore, the estimated values of $\gamma_1$ and $\gamma_2$ are *both* quite small, implying via Equation (5) that the money demand shock, $\hat{e}_t$, must account for nearly all the movements in real balances, $\hat{m}_t$. In this case, apparently, the estimation procedure continues to prefer a version of the model in which movements in real balances have little effect on the dynamics of output and inflation; but with $\omega_2$ constrained to be nonzero and large, this goal is accomplished in a roundabout way by forcing movements in shift-adjusted money, $\hat{m}_t - \hat{e}_t$, to always be small! Once again, these results point to the statistical adequacy of popular specifications, in which real balances do not enter the IS and Phillips curves.

Figure 1 bolsters this conclusion by plotting the impulse response of each variable—detrended output, detrended real balances, inflation, and the interest rate—to three of the model's shocks: the preference shock, $\varepsilon_{at}$, the technology shock, $\varepsilon_{zt}$, and the policy shock, $\varepsilon_{rt}$. The figure omits the impulse responses to the money demand shock, $\varepsilon_{et}$, since, as explained above, these responses are trivial: under the interest rate rule (Equation 9), money demand shocks are fully accommodated by shifts in the money supply, leaving output and inflation unchanged.

In Figure 1, the impulse responses of output and inflation implied by the constrained model with $\omega_2 = 0.25$ coincide almost exactly with those implied by the unconstrained model with $\omega_2 = 0$. These results reflect the fact that when $\omega_2$ is forced to be large, the estimation procedure compensates with settings for $\gamma_1$ and $\gamma_2$ that imply much smaller movements in shift-adjusted real balances, $\hat{m}_t - \hat{e}_t$, as can be seen by comparing the very different impulse responses for real balances that emerge from the constrained and unconstrained models. More generally, the impulse responses displayed in Figure 1 look quite reasonable. A one standard deviation IS shock, $\varepsilon_{at}$, increases output by about 0.5% and increases the annualized inflation rate by about 100 basis points; under the estimated policy rule, the monetary

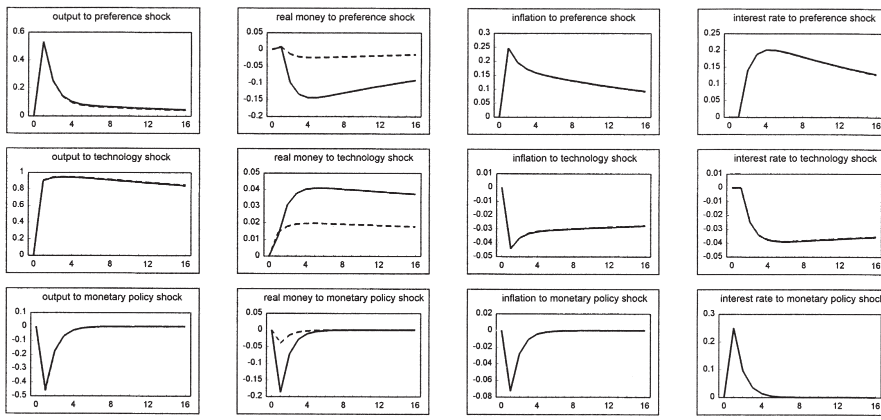


FIG. 1. Impulse responses. Each panel shows the percentage-point response of one of the model's variables to a one standard deviation shock. Solid lines correspond to the unconstrained model; dashed lines correspond to the constrained model.

authority responds with an 80 basis-point increase in the annualized nominal interest rate. A one standard deviation technology shock, $\varepsilon_{zt}$, increases output by nearly 1%, decreases the annualized inflation rate by 17 basis points, and induces the monetary authority to lower the annualized interest rate by 15 basis points. Finally, a one standard deviation policy shock, $\varepsilon_{rt}$, corresponds to a 100 basis-point tightening that decays over four or five quarters, generating a 0.4% fall in output and a 29 basis-point fall in annualized inflation.

Overall, therefore, the results obtained here indicate that the post-1980 U.S. data prefer standard specifications, such as those developed by Rotemberg and Woodford (1997) and McCallum and Nelson (1999), in which measures of money are absent from the IS and Phillips curves, to alternatives in which money plays a more important role in the monetary business cycle. Moreover, these results appear quite robust to changes in the estimation strategy. Ireland (2001), for instance, obtains small estimates of the key parameter, $\omega_2$, even when the risk aversion and price adjustment cost parameters, $\omega_1$ and $\psi$, are estimated together with the rest of the model, instead of calibrated as they are here. And, as Table 1 also shows, the basic results remain unchanged when M1 replaces M2 as the measure of money in the data.

## 3. CONCLUSIONS, COMPARISONS, AND CAVEATS

Maximum likelihood estimation of a small, structural model of the business cycle suggests that real balances fail to enter into the IS and Phillips curve equations that govern the dynamics of output and inflation. Fuhrer (1994), working with Fuhrer and Moore's (1995) forward-looking model without microfoundations, and Rudebusch and Svensson (2002), working with their backward-looking model without microfoundations, report similar results. The microfounded model used here, however, implies that empirical measures of real balances must be adjusted for shifts in money demand in order to accurately isolate and quantify the direct effects that changes in money have on output and inflation. Thus, the results obtained here generalize those found previously: even after correcting for money demand shocks, money's role in the monetary business cycle appears limited.

McCallum (2000) and Woodford (1999) calibrate forward-looking, microfounded models that are similar to the one used here. Both start by assigning values to the risk aversion parameter, $\omega_1$, and the money demand parameters, $\gamma_1$ and $\gamma_2$, then back out an implied value for $\omega_2$, the key parameter that determines whether or not real balances belong in the IS and Phillips curves, using cross-equation restrictions like Equation (7) from above. This procedure leads McCallum to set $\omega_2 = 0.0199$ and Woodford to choose the much larger value $\omega_2 = 0.1$. Here, more formal econometric methods identify a value for $\omega_2$ that is even smaller than McCallum's, despite the fact that this smaller estimate of $\omega_2$ requires what might be considered a less reasonable value for $\gamma_1$, measuring the income elasticity of money demand.

Of course, these conclusions must be accompanied by several caveats. First, the data used here cover just one episode from U.S. monetary history; it would certainly be useful to investigate whether similar results can be obtained with data from other periods and other countries. Along these lines, recent work by Andres, Lopez-Salido, and Valles (2001) repeats the estimation exercise performed here, but with post-1980 data from the Euro area. They, too, find little evidence of a direct role for money in the IS and Phillips curve equations.

Second, while general in some respects, the small, structural model used here remains quite stylized along many dimensions. As emphasized by Nelson (2000), larger and more complicated models may serve to highlight alternative channels through which changes in money affect the economy, and future research must explore this possibility. In addition, Fuhrer (2000) and Roberts (1997) show that the empirical performance of purely forward-looking models of nominal price rigidity can be improved significantly through the introduction of backward-looking elements in the IS and Phillips curves. Similarly, Gali and Gertler (1999) and Sbordone (2002) argue that forward-looking models provide a closer fit to the data when labor market rigidities are introduced as well. All of these studies suggest that it would be worthwhile to extend the research project initiated here, by testing for the significance of money's role in models that are more elaborate—and hence more empirically successful—than the one used here.

Finally, it must be noted that the results obtained here, while implying that money plays a minimal role in shaping the dynamic behavior of output and inflation, do not excuse the monetary authority from the important task of controlling inflation. In the model, the monetary authority must choose the steady-state rate of inflation, which is ultimately determined by the rate of nominal money growth, exactly as described by the quantity theory of money. And in the model, the monetary authority must limit the variability of inflation by vigorously adjusting its interest rate instrument in response to changes in a nominal anchor, exactly as described by Parkin (1978), McCallum (1981), Kerr and King (1996), and Clarida, Gali, and Gertler (2000).

## APPENDIX

This appendix shows how Equations (4–6) summarize the optimizing behavior of the representative household and intermediate goods-producing firm. It also shows how the parameters that enter into Equations (4–6) ultimately depend on the underlying parameters describing private agents' tastes and technologies.

The representative household chooses $c_t$, $h_t$, $B_t$, and $M_t$ for all $t = 0,1,2,...$ to maximize its expected utility, subject to its budget constraints. The first-order conditions for this problem can be written as

$$\eta = u_1(c_t, m_t/e_t)w_t \,, \tag{A1}$$

$$a_t u_1(c_t, m_t/e_t) = \beta r_t E_t[a_{t+1}u_1(c_{t+1}, m_{t+1}/e_{t+1})/\pi_{t+1}] \,, \tag{A2}$$

$$r_t u_2(c_t, m_t/e_t) = (r_t - 1)e_t u_1(c_t, m_t/e_t) \,, \tag{A3}$$

and

$$w_t h_t + d_t = c_t \tag{A4}$$

for all $t = 0,1,2,....$ In Equations (A1–A3), $u_1$ and $u_2$ denote the derivatives of the utility function, $u$, with respect to its first and second arguments, while $w_t = W_t/P_t$ denotes the real wage. Equation (A4) is derived from the household's budget constraint when the market-clearing conditions $M_t = M_{t-1} + T_t$ and $B_t = B_{t-1} = 0$ are imposed; $d_t = D_t/P_t$ denotes real dividends.

The representative intermediate goods-producing firm chooses $P_t(i)$ for all $t = 0,1,2,...$ to maximize its total market value, given by

$$E \sum_{t=0}^{\infty} \beta^t a_t u_1(c_t, m_t/e_t)[D_t(i)/P_t] \,,$$

where $\beta^t\, a_t u_1(c_t,m_t/e_t)$ measures the marginal utility value to the representative household of an additional dollar in profits received during period $t$ and where

$$\frac{D_t(i)}{P_t} = \left[\frac{P_t(i)}{P_t}\right]^{1-\theta} y_t - \left[\frac{P_t(i)}{P_t}\right]^{-\theta}\left(\frac{w_t y_t}{z_t}\right) - \frac{\phi}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right]^2 y_t \tag{A5}$$

for all $t = 0,1,2,....$ The expression (Equation A5) for the firm's real dividend payment incorporates the linear production function along with the requirement that the firm supply output on demand; it also shows how the cost of price adjustment subtracts from profits. The first-order conditions for this problem are

$$0 = (1-\theta)\left[\frac{P_t(i)}{P_t}\right]^{-\theta}\left(\frac{y_t}{P_t}\right)$$

$$+ \theta\left[\frac{P_t(i)}{P_t}\right]^{-\theta-1}\left(\frac{y_t w_t}{z_t P_t}\right) - \phi\left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right]\left[\frac{y_t}{\pi P_{t-1}(i)}\right]$$

$$+ \beta\phi E_t\left\{\left[\frac{a_{t+1}u_1(c_{t+1}, m_{t+1}/e_{t+1})}{a_t u_1(c_t, m_t/e_t)}\right]\left[\frac{P_{t+1}(i)}{\pi P_t(i)} - 1\right]\left[\frac{y_{t+1}P_{t+1}(i)}{\pi P_t(i)^2}\right]\right\} \tag{A6}$$

for all $t = 0,1,2,\ldots$.

In a symmetric equilibrium, where $y_t(i) = y_t$, $h_t(i) = h_t$, $P_t(i) = P_t$, $D_t(i) = D_t$, and $y_t = z_t h_t$ for all $i \in [0,1]$ and $t = 0,1,2,\ldots$, Equations (A4) and (A5) can be combined to derive the economy's aggregate resource constraint,

$$y_t = c_t + \frac{\phi}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 y_t\,, \tag{A7}$$

while Equations (A1) and (A6) can be combined to yield

$$\theta - 1 = \theta\left[\frac{\eta}{z_t u_1(c_t, m_t/e_t)}\right] - \phi\left(\frac{\pi_t}{\pi} - 1\right)\left(\frac{\pi_t}{\pi}\right)$$

$$+ \beta\phi E_t\left\{\left[\frac{a_{t+1}u_1(c_{t+1}, m_{t+1}/e_{t+1})}{a_t u_1(c_t, m_t/e_t)}\right]\left(\frac{\pi_{t+1}}{\pi} - 1\right)\left(\frac{y_{t+1}}{y_t}\right)\left(\frac{\pi_{t+1}}{\pi}\right)\right\}. \tag{A8}$$

Equations (A2), (A3), (A7), and (A8) imply that in the absence of shocks, the economy converges to a steady state, in which $y_t = y$, $c_t = c$, $m_t = m$, $\pi_t = \pi$, and $r_t = r$. The monetary authority must choose the steady-state inflation rate, $\pi$; Equation (A2) then requires that $r = \pi/\beta$. Equation (A7) reveals that the cost of price adjustment is zero in the steady state, so that $c = y$. Together, Equations (A3) and (A8) determine $y$ and $m$ as solutions to the two equations

$$ru_2(y, m/e) = (r - 1)eu_1(y, m/e)$$

and

$$(\theta - 1)zu_1(y, m/e) = \theta\eta.$$

Now let $\hat{y}_t = \ln(y_t/y)$, $\hat{c}_t = \ln(c_t/c)$, $\hat{m}_t = \ln(m_t/m)$, $\hat{\pi}_t = \ln(\pi_t/\pi)$, $\hat{r}_t = \ln(r_t/r)$, $\hat{a}_t = \ln(a_t)$, $\hat{e}_t = \ln(e_t/e)$, and $\hat{z}_t = \ln(z_t/z)$, as described in the text. A first-order Taylor approximation to Equation (A7) around the model's steady state implies that $\hat{c}_t = \hat{y}_t$. Hence, first-order approximations to Equations (A2), (A3), and (A8) can be written as Equations (4–6) in the text, where

$$\omega_1 = -\frac{u_1(y, m/e)}{yu_{11}(y, m/e)}\,, \tag{A9}$$

$$\omega_2 = -\frac{(m/e)u_{12}(y, m/e)}{yu_{11}(y, m/e)}\,, \tag{A10}$$

$$\gamma_2 = \frac{r}{(r-1)(m/e)}\left[\frac{u_2(y, m/e)}{(r-1)eu_{12}(y, m/e) - ru_{22}(y, m/e)}\right], \tag{A11}$$

and

$$\psi = (\theta - 1)/\phi \tag{A12}$$

and where $\gamma_1$ and $\gamma_3$ are determined by Equations (7) and (8) in the text.

In Equations (A9–A11), the $u_{ij}$s denote the second derivatives of the utility function, $u$. Thus, in particular, Equation (A9) shows that $\omega_1$ depends inversely on the household's relative risk aversion. Equation (A10) indicates that $\omega_2 > 0$, so that changes in real balances enter into the IS and Phillips curves, if and only if $u_{12} > 0$, so that utility is nonseparable across consumption and real balances. Finally, Equation (A12) reveals that the parameter $\psi$ in the Phillips curve (Equation 6) is inversely related to the cost-of-price-adjustment parameter, $\phi$.

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