

Revisiting the uncertain unit root in GDP and CPI: Testing for non-linear trend reversion

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Abstract

We test for the presence of a unit root in U.S. GDP and CPI, allowing for non-linear trend reversion under the alternative hypothesis. In contrast to most previous results, we find evidence in favour of trend stationarity for both variables.

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1. Introduction

The long-held trend-stationary view of macroeconomic variables was challenged by Nelson and Plosser's (1982) seminal paper arguing that many U.S. macroeconomic time series were best characterised as unit-root processes. Since then, the time-series properties of real output, money aggregates, prices and wages have received much attention, with numerous unit-root tests applied to the data and the properties of those tests heavily discussed. Empirical findings have been divided, unable to provide unified support for one theoretical view over another. Perron (1989) and Evans (1989), for example, provided some evidence in support of the unit-root model, only to be subsequently contradicted by findings of trend stationarity presented by Clark (1989), Ben-David and Papell (1995), Diebold and Senhadji (1996) and Papell and Prodan (2004).

Most unit-root tests rely upon a linear framework, with Dickey–Fuller-type tests among the most commonly employed. However, when the alternative is an autoregressive parameter

near unity, which must be seen as the empirically relevant case when testing common macroeconomic variables, the power of Dickey–Fuller-type tests is known to be low (Froot and Rogoff, 1995). Sercu et al. (1995) and Michael et al. (1997) have also noted that the Augmented Dickey–Fuller test (Said and Dickey, 1984) lacks power against stationary alternatives if the underlying model is exponential smooth-transition autoregressive (ESTAR) rather than standard linear autoregressive. In an ESTAR model, the speed of mean or trend reversion to the equilibrium is not constant; rather, the process can display unit-root behaviour in the region close to its equilibrium but strong reversion when the process is far from its mean or trend.

We believe that an ESTAR process is a plausible characterisation of price and output dynamics. For instance, an ESTAR process for the consumer price index (CPI) would be consistent with a central bank that reacts very little – or not at all – to small deviations from its target level but strongly to large disequilibria. This can be motivated either by the view that close to target, fine-tuning is hindered by the uncertain lags of monetary-policy and data-measurement errors, or by the idea that a central bank observing a target band faces a non-linearly rising penalty as the target variable nears the tolerated edges, as described in Mishkin and Westelius (2006). Such non-linear responses to disequilibria may, as a result, impart non-linearity

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Table 1
Unit-root tests for trend stationarity of GDP and CPI

	CPI	GDP
ADF	−1.728	−2.842
ADF-GLS	−1.572	−2.181
KSS	−3.652 ^a	−3.756 ^a
Sample	January 1914 to July 2005	1947 Q1 to 2005 Q2

Notes: ADF is the test statistic from the Augmented Dickey–Fuller test, ADF-GLS is the test statistic from the Augmented Dickey–Fuller test with GLS detrending and KSS is the test statistic from the KSS test.

^a Denotes significant result at the 5% level.

to the dynamics of real variables. In such environments, modelling reversion with a constant speed of adjustment as in the linear autoregressive model is a poor approximation. In this paper, we apply the Kapetanios et al. (2003) ESTAR unit-root test (KSS test) to long series of U.S. consumer prices and real GDP and compare this test to two traditional unit-root tests — the Augmented Dickey–Fuller (ADF) test and ADF test with GLS detrending (Elliott et al., 1996) (ADF-GLS).

2. ESTAR and linear unit-root tests: empirical results

The KSS test pits the null hypothesis of a unit-root process against the alternative of a globally stationary ESTAR process. Whilst the data-generating process of the ESTAR model is complex (Granger and Teräsvirta, 1993), the KSS test is straightforward to perform using OLS-detrended values (x_t) of the original time series (\tilde{x}_t). Denoting the OLS estimators of the intercept and trend as $\hat{\mu}$ and $\hat{\psi}t$ respectively, define

$$x_t = \tilde{x}_t - \hat{\mu} - \hat{\psi}t, \quad (1)$$

and perform the regression

$$\Delta x_t = \delta x_{t-1}^3 + \sum_{i=1}^p \gamma_i \Delta x_{t-i} + \varepsilon_t \quad (2)$$

where $\varepsilon_T \sim iid(0, \sigma_\varepsilon^2)$.¹ The null hypothesis $H_0: \delta = 0$ is then tested versus $H_1: \delta < 0$ using the t -statistic on $\hat{\delta}$. The null hypothesis of a unit root is rejected for small enough values of the test statistic; critical values can be found in Kapetanios et al. (2003).

We use the quarterly, seasonally-adjusted level of real U.S. GDP (total, chain weighted) from 1947 Q1 to 2005 Q2 and the monthly level of the CPI (all items) from January 1914 to July 2005 (from the BEA and BLS respectively). Both series are transformed into natural logarithms for the analysis. Lag length for the ADF and ADF-GLS tests are determined by the Akaike information criterion and the lag length in the KSS test regressions, p , is set equal to that of the ADF test. This is a reasonable approach, for as pointed out by Kapetanios et al., linear dynamics can be seen as a first order approximation if the true augmentations are non-linear in nature.

When fully-parameterised ESTAR models are estimated, pre-testing is typically used to establish that the time series of interest is non-linear. This will not be done here as the problem of potential non-identification of the parameters of the ESTAR model is circumvented through the design of the test; see Kapetanios et al. (2003) for details. Moreover, pre-testing is neither feasible, since currently available tests of non-linearity are derived assuming stationarity of the series in question, nor interesting, since the test has the correct size if the time series is in fact linear.

The results of the three unit-root tests are reported in Table 1. The traditional unit-root tests provide no evidence of trend-stationary, failing to reject the unit-root null at conventional significance levels. In contrast, the KSS test rejects the null of a unit root in both GDP and CPI at the 5% level, thereby favouring a globally stationary ESTAR process for both series.

The KSS test results have several implications, first and foremost giving credence to the trend-stationary view of the world, albeit with non-linear dynamics. The finding of trend-stationary real GDP implies that output should be decomposed using a time-trend regression into a trend component and a stationary cyclical component. Moreover, the forecast-error variance will be bounded even in the limit. The finding that CPI is stationary is, in our opinion, more interesting. Trend-stationary CPI is consistent with *price-level targeting* and not *inflation targeting*, as is more commonly assumed in the monetary-policy literature.² This distinction is crucial for evaluating central bank behaviour and stabilisation. Under an inflation-targeting regime, long-run expectations about the price level are poorly anchored, as the price level will be generated by a unit-root process with drift. Price-level targeting, in contrast, anchors expectations about the price level. The rejection of the KSS null, which is consistent with the Federal Reserve pursuing price-level targeting over a long sample, suggests that some modelling choices in applied monetary-policy research may need to be reconsidered.

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¹ The value of the delay parameter has been set to $d=1$ in this application. Similar to Taylor et al. (2001), we argue that the delay parameter should be small as we expect reversion to begin quickly.

² For a theoretical discussion of price-level and inflation targeting, see Svensson (1999) and Vestin (2006).

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